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**Pearson Edexcel
Level 3 GCE**

Centre Number

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Candidate Number

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Further Mathematics

Advanced

Further Mathematics Option 1

Paper 3: Further Pure Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 1 hour 30 minutes

Paper Reference

9FM0/3A

You must have:

Mathematical Formulae and Statistical Tables, calculator

Total Marks

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Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1. Use Simpson's Rule with 6 intervals to estimate

$$\int_1^4 \sqrt{1+x^3} dx$$

(5)

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2. Given k is a constant and that

$$y = x^3 e^{kx}$$

use Leibnitz theorem to show that

$$\frac{d^n y}{dx^n} = k^{n-3} e^{kx} (k^3 x^3 + 3nk^2 x^2 + 3n(n-1)kx + n(n-1)(n-2)) \quad (4)$$

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Question 2 continued

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(Total for Question 2 is 4 marks)

3. A vibrating spring, fixed at one end, has an external force acting on it such that the centre of the spring moves in a straight line. At time t seconds, $t \geq 0$, the displacement of the centre C of the spring from a fixed point O is x micrometres.

The displacement of C from O is modelled by the differential equation

$$t^2 \frac{d^2x}{dt^2} - 2t \frac{dx}{dt} + (2 + t^2)x = t^4 \quad (\text{I})$$

- (a) Show that the transformation $x = tv$ transforms equation (I) into the equation

$$\frac{d^2v}{dt^2} + v = t \quad (\text{II})$$

(5)

- (b) Hence find the general equation for the displacement of C from O at time t seconds.

(7)

- (c) (i) State what happens to the displacement of C from O as t becomes large.

- (ii) Comment on the model with reference to this long term behaviour.

(2)

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Question 3 continued

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(Total for Question 3 is 14 marks)

4.
$$\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + y = 0 \quad (I)$$

(a) Show that

$$\frac{d^5y}{dx^5} = ax \frac{d^4y}{dx^4} + b \frac{d^3y}{dx^3}$$

where a and b are integers to be found.

(4)

(b) Hence find a series solution, in ascending powers of x , as far as the term in x^5 , of the differential equation (I) where $y = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$

(5)

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5. The normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ passes through the parabola again at the point $Q(aq^2, 2aq)$.

The line OP is perpendicular to the line OQ , where O is the origin.

Prove that $p^2 = 2$

(9)

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Question 5 continued

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(Total for Question 5 is 9 marks)

6. A tetrahedron has vertices $A(1, 2, 1)$, $B(0, 1, 0)$, $C(2, 1, 3)$ and $D(10, 5, 5)$.

Find

- (a) a Cartesian equation of the plane ABC . (3)

- (b) the volume of the tetrahedron $ABCD$. (3)

The plane Π has equation $2x - 3y + 3z = 0$

The point E lies on the line AC and the point F lies on the line AD .

Given that Π contains the point B , the point E and the point F ,

- (c) find the value of k such that $\vec{AE} = k\vec{AC}$. (3)

Given that $\vec{AF} = \frac{1}{9}\vec{AD}$

- (d) show that the volume of the tetrahedron $ABCD$ is 45 times the volume of the tetrahedron $ABEF$. (2)

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7. P and Q are two distinct points on the ellipse described by the equation $x^2 + 4y^2 = 4$

The line l passes through the point P and the point Q .

The tangent to the ellipse at P and the tangent to the ellipse at Q intersect at the point (r, s) .

Show that an equation of the line l is

$$4sy + rx = 4 \quad (8)$$

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Question 7 continued

(Total for Question 7 is 8 marks)

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8.

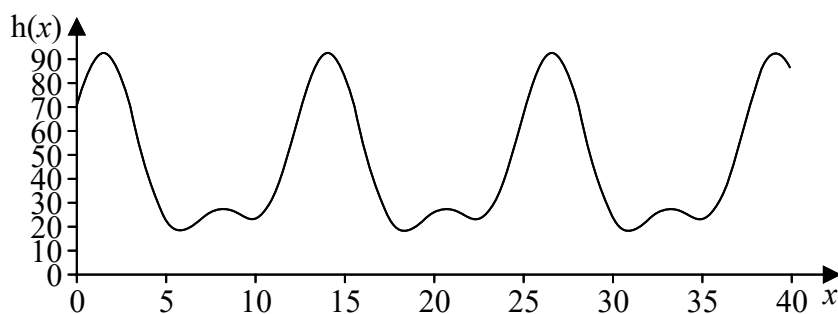


Figure 1

Figure 1 shows the graph of the function $h(x)$ with equation

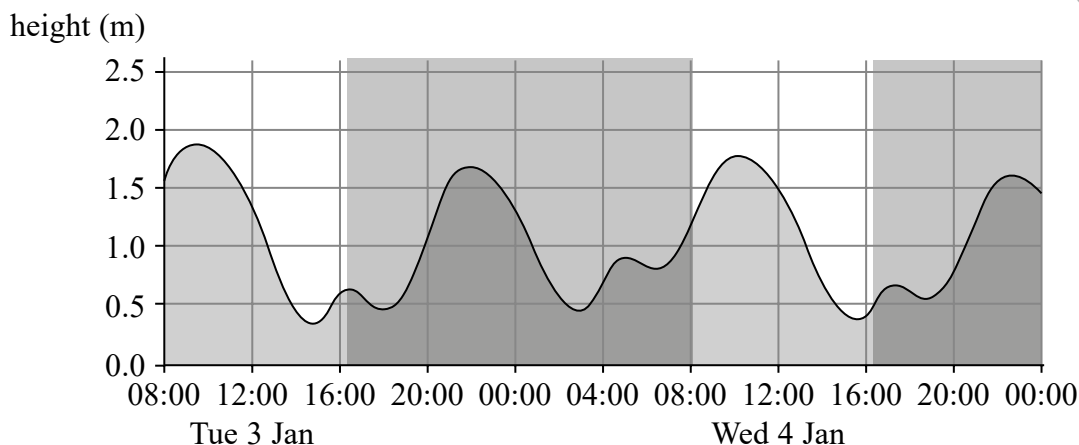
$$h(x) = 45 + 15 \sin x + 21 \sin\left(\frac{x}{2}\right) + 25 \cos\left(\frac{x}{2}\right) \quad x \in [0, 40]$$

(a) Show that

$$\frac{dh}{dx} = \frac{(t^2 - 6t - 17)(9t^2 + 4t - 3)}{2(1 + t^2)^2}$$

where $t = \tan\left(\frac{x}{4}\right)$.

(6)



Source: ¹Data taken on 29th December 2016 from <http://www.ukho.gov.uk/easytide/EasyTide>

Figure 2

Figure 2 shows a graph of predicted tide heights, in metres, for Portland harbour from 08:00 on the 3rd January 2017 to the end of the 4th January 2017¹.

The graph of $kh(x)$, where k is a constant and x is the number of hours after 08:00 on 3rd of January, can be used to model the predicted tide heights, in metres, for this period of time.

(b) (i) Suggest a value of k that could be used for the graph of $kh(x)$ to form a suitable model.

(ii) Why may such a model be suitable to predict the times when the tide heights are at their peaks, but not to predict the heights of these peaks?

(3)

(c) Use Figure 2 and the result of part (a) to estimate, to the nearest minute, the time of the highest tide height on the 4th January 2017.

(6)

Question 8 continued

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Question 8 continued

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(Total for Question 8 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS